Practice Set 1.2

1) Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm.

→ Here $\theta = 108°$

$\therefore \theta = \left(108 \times \frac{\pi}{180}\right)^c = \frac{3\pi}{5}^c \quad \therefore \boxed{\theta = \frac{3\pi}{5}^c}$

Radius $(r) = 15$ cm

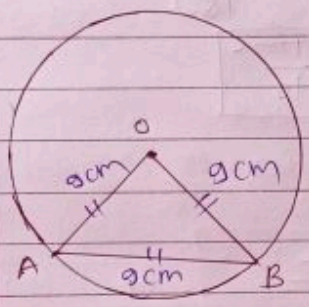
$\therefore$ length of an arc $(s) = r\theta$

$= 15 \times \frac{3\pi}{5}$

$= \underline{9\pi \text{ cms}}$

2) The radius of a circle is 9cm. Find the length of an arc of this circle which cuts of a chord of length, equal to length of radius.

→



In figure radius = AO = BO = 9cm

$\therefore$ Radius = length of chord

$\therefore$ AB = AO = BO = 9cm

$\therefore$ $\triangle$ AOB is equaleteral triangle

$\therefore$ $m\angle AOB = 60°$

$\therefore \theta = 60°$

$\therefore \theta = \left(60 \times \frac{\pi}{180}\right)^c$

$\boxed{\theta = \frac{\pi}{3}^c} \qquad \boxed{r = 9\text{cm}}$

$\therefore$ length of an arc $(s) = r\theta$

$= 9 \times \frac{\pi}{3}$

$= \underline{3\pi \text{ cm}}$

3) A pen Find the angle in degree subtended at the centre of circle by an arc whose length is 15 cm, if the radius of the circle is 25 cm

→ Here, length of an arc $(s) = 15$ cm and $\boxed{r = 25\text{cm}}$

Find $\theta = ?$

$\therefore s = r\theta$

$15 = 25 \times \theta$

$\frac{15}{25} = \theta$

$\boxed{\theta = \frac{3}{5}^c}$

$\theta = \left(\frac{3}{5} \times \frac{180}{\pi}\right)^\circ$

$= \frac{540}{5\pi}$

$= \frac{108}{\pi}$

$= \frac{108}{\frac{22}{7}}$

$= \frac{108 \times 7}{22}$

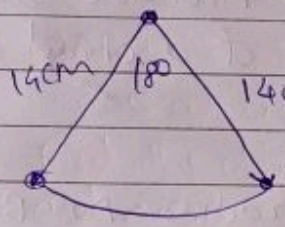
$\theta = \frac{54 \times 7}{11}$

$\theta = \frac{378}{11}$

$\boxed{\theta = 34.40^\circ}$

4) A pendulum of length 14cm oscillates through an angle of 18°. Find the length of its path

→



Here $\theta = 18^\circ$

$= 18 \times \frac{\pi}{180}$

$\boxed{\theta = \frac{\pi}{10}^c}$

radius $(r) = 14$ cm

length of path $(s) = r \times \theta$

$= 14 \times \frac{\pi}{10}$

$= \frac{7\pi}{5}$

$= \frac{7 \times \frac{22}{7}}{5}$

$= \frac{22}{5}$

$= 4.4$ cm

∴ the length of path of pendulum = 4.4cm

5) Two arcs of the same length subtended angles of 60° and 75° at the centre of two circles. What is the ratio of ~~two circles~~ radius of two circles?

→ Measure of arcs of circle whose subtended angle ($\theta_1 = 60°$) is s_1 and radius is r_1

Measure of arcs of circle whose subtended angle ($\theta_2 = 75°$) is s_2 and radius is r_2

$\therefore \theta_1 = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi}{3}^c \quad \therefore \boxed{\theta_1 = \frac{\pi}{3}^c}$

$\theta_2 = 75° = \left(75 \times \frac{\pi}{180}\right)^c = \frac{5\pi}{12}^c \quad \therefore \boxed{\theta_2 = \frac{5\pi}{12}^c}$

$s_1 = r_1\theta_1 = r_1 \times \frac{\pi}{3} = \frac{\pi}{3} r_1 \quad \therefore \boxed{s_1 = \frac{\pi}{3} r_1}$

$s_2 = r_2\theta_2 = r_2 \times \frac{5\pi}{12} = \frac{5\pi}{12} r_2 \quad \therefore \boxed{s_2 = \frac{5\pi}{12} r_2}$

Hence, $s_1 = s_2$

$$\frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$$

$$\frac{r_1}{r_2} = \frac{5}{12} \times \frac{3}{4}$$

$$\frac{r_1}{r_2} = \frac{5}{4}$$

$$r_1 : r_2 = 5 : 4$$

$\therefore$ the ratio of radius of two circles = 5:4

6) The area of a circle is 25π sq.cm. Find the length of its arc subtending an angle of 144° at the centre. Also find the area of the corresponding sector

→ Area of circle = 25π sq.cm

$\therefore \pi r^2 = 25\pi$

$r^2 = 25$

$\boxed{r = 5 \text{ cm}}$

$\theta = 144° = \left(144 \times \frac{\pi}{180}\right)^c = \frac{4\pi}{5}^c \quad \boxed{\theta = \frac{4\pi}{5}^c}$

$r = 5\text{cm} \quad \theta = \frac{4\pi}{5}^c$

$\therefore$ length of an arc $= r\theta$

$= 5 \times \frac{4\pi}{5}$

$= 4\pi\text{cm}$

Area of sector $= \frac{1}{2} rs$

$= \frac{1}{2} \times 5 \times 4\pi$

$= \frac{1}{2} \times 20\pi$

$= 10\pi$ Sq. cm

7) OAB is a sector of the circle circle having centre at O and radius 12cm. If $m\angle AOB = 45°$, Find the diff. between the area of sector OAB and $\triangle AOB$

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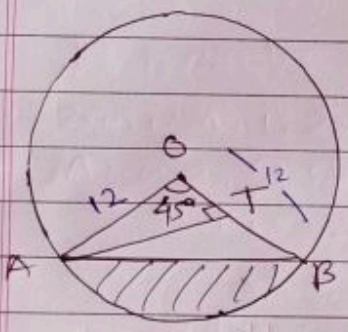

$m\angle AOB\ (\theta) = 45°$

$= \left(45 \times \frac{\pi}{180}\right)^c$

$\theta = \frac{\pi}{4}\ c$ $\quad r = 12\text{cm}$

$\therefore$ Area of sector OAB (A_1)

$= \frac{1}{2} r^2\theta$

$= \frac{1}{2} \times 12 \times 12 \times \frac{\pi}{4}$

$= 18\pi$ Sq. cm

In $\triangle AOB$, $\angle AOB = 45°$ Draw $AT \perp OB$

In $\triangle ATO$, $\angle AOB = 45°$

$\angle ATO = 90°$

$\angle OAB = 45°$

$\therefore$ by $45°-45°-90°$ Theorem

$AT = \frac{1}{\sqrt{2}} \times OA = \frac{1}{\sqrt{2}} \times 12 = \frac{12}{\sqrt{2}}$

∴ In $\triangle AOB$, Base $= OB = 12$ cm

height $= AT = \frac{12}{\sqrt{2}}$ cm

$\therefore A(\triangle AOB)\ (A_2) = \frac{1}{2} \times OB \times AT$

$= \frac{1}{2} \times 12^{6} \times \frac{12}{\sqrt{2}}$

$= \frac{72}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{72\sqrt{2}}{2}$

$= 36\sqrt{2}$

∴ Diff. betwee A(O-AB) and A($\triangle$OAB)

$= A_1 - A_2$

$= 18\pi - 36\sqrt{2}$

$= 18(\pi - 2\sqrt{2})$ sq. cm

8) OPQ is a sector of a circle having Centre O and radius is 15 cm. If $m\angle POQ = 30°$. Find area enclosed by arc PQ and chord PQ

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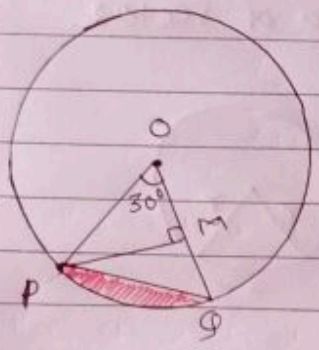


Here $m\angle POQ = \theta = 30°$

$= 30 \times \frac{\pi}{180}$

$\boxed{\theta = \frac{\pi}{6}^{c}}$

In $\triangle POQ$, $\angle POQ = 30°$ $\boxed{OP = OQ = 15 cm}$

Draw $PM \perp OQ$

∴ In $\triangle PMO$, $\angle M = 90°$, $\angle O = 30°$

$\therefore \angle OPM = 60°$

∴ by 30°-60°-90° property / Theorem

$\therefore PM = \frac{1}{2} OP$ ... (Side opp. 30°)

$= \frac{1}{2} \times 15$

$\boxed{PM = \frac{15}{2} cm}$

$\therefore A(\triangle POQ) = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times OQ \times PM$

$= \frac{1}{2} \times 15 \times \frac{15\sqrt{3}}{2}$

$= \frac{225\sqrt{3}}{4}$

$\therefore$ Area of sector $O-PQ = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 15^2 \times \frac{\pi}{6}$

$= \frac{225\pi}{12}$

$\therefore$ Area enclosed by arc PQ and chord PQ

= Area of sector O-PQ - A($\triangle$ POQ)

$= \frac{225\pi}{12} - \frac{225\sqrt{3}}{4}$

$= \frac{225}{4}\left(\frac{\pi}{3} - \sqrt{3}\right)$ sq.cm.

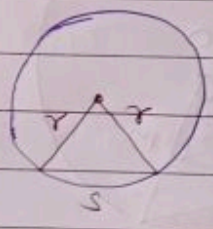
9) The perimeter of a sector of a circle of area 25π sq.cm is 20cm. Find the area of sector.

→ Area of circle = 25π sqcm

$\therefore \pi r^2 = 25\pi$

$r^2 = 25$

$\boxed{r = 5 \text{ cm}}$



perimeter of sector = 20cm

$\therefore r + r + S = 20\text{cm}$ ---(S = length of an arc)

$\therefore 5 + 5 + S = 20$

$10 + S = 20$

$S = 20 - 10$

$\boxed{S = 10\text{cm}}$

$\therefore$ Area of sector $= \frac{1}{2} rs$

$= \frac{1}{2} \times 5 \times 10$

$= 5 \times 5$

$= 25$ sq.cm

10) The perimeter of the sector of circle of area 64π sq cm is 56 cm. Find the area of sector.

→ Area of circle $= 64\pi$ sq·cm

$\therefore \pi r^2 = 64\pi$

$r^2 = 64$

$\boxed{r = 8 \text{ cm}}$

$\therefore$ perimeter of circle $= 56$ cm

$\therefore r + r + S = 56$ ... (S = Length of an arc)

$\therefore 8 + 8 + S = 56$

$\therefore 16 + S = 56$

$S = 56 - 16$

$\boxed{S = 40 \text{ cm}}$

Area of sector $= \frac{1}{2} rS$

$= \frac{1}{2} \times 8 \times 40$

$= 4 \times 40$

$= 160$ sq·cm